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# An Evaluation of a Coupled Microstructural Approach for the Analysis of Functionally Graded Composites via the Finite-Element Method

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**AN EVALUATION OF A COUPLED MICROSTRUCTURAL APPROACH  
FOR THE ANALYSIS OF FUNCTIONALLY GRADED COMPOSITES  
VIA THE FINITE-ELEMENT METHOD**

by

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**ABSTRACT**

A comparison is presented between the predictions of the finite-element analysis and a recently developed higher-order theory for functionally graded materials subjected to a through-thickness temperature gradient. In contrast to existing micromechanical theories that utilize classical (i.e., uncoupled) homogenization schemes to calculate micro-level and macro-level stress and displacement fields in materials with uniform or nonuniform fiber spacing (i.e., functionally graded materials), the new theory explicitly couples the microstructural details with the macrostructure of the composite. Previous thermo-elastic analysis has demonstrated that such coupling is necessary when: the temperature gradient is large with respect to the dimension of the reinforcement; the characteristic dimension of the reinforcement is large relative to the global dimensions of the composite and the number of reinforcing fibers or inclusions is small. In these circumstances, the standard micromechanical analyses based on the concept of the representative volume element used to determine average composite properties produce questionable results. The comparison between the predictions of the finite-element method and the higher-order theory presented herein establish the theory's accuracy in predicting thermal and stress fields within composites with a finite number of fibers in the thickness direction subjected to a through-thickness thermal gradient.

**INTRODUCTION**

The past thirty years have seen tremendous growth in the development and use of composite materials. The applications range from sporting and recreational accessories to advanced aerospace structural and engine components. Traditionally, the reinforcement phase in different types of composite materials is distributed uniformly such that the resulting mechanical, thermal or physical properties do not vary spatially. Recently, a new concept involving tailoring or engineering the internal microstructure of a composite material to specific applications has taken root. This concept involves spatially varying the microstructural details through nonuniform distribution of the reinforcement phase, by using reinforcement with different properties, sizes and shapes, as well as by interchanging the roles of reinforcement and matrix phases in a continuous

manner. Hence the name functionally gradient materials (FGMs) coined by Japanese researchers to describe these newly emerging materials. The result is a microstructure that produces continuously changing thermal and mechanical properties at the macroscopic or continuum level.

Functionally graded composites are ideal candidates for applications involving severe thermal gradients, ranging from thermal structures in advanced aircraft and aerospace engines to computer circuit boards. In such applications, ceramic-rich region of a functionally graded composite is exposed to hot temperature while metallic-rich region is exposed to cold temperature, with a gradual microstructural transition in the direction of the temperature gradient. By adjusting the microstructural transition appropriately, optimum temperature distribution can be realized. Microstructural grading can also be effectively used to reduce the mismatch in the thermo-mechanical properties between differently oriented, adjacent plies in a laminated plate. Thus reduction of the interlaminar stresses at the free edge of a laminate that result from a large property mismatch between adjacent plies can be realized by using the functional grading concept to smooth out the transition between dissimilar plies. Along similar lines, joining of dissimilar materials can be made more efficient through the use of functionally graded joints. Other benefits to be realized from the use of functionally graded architectures include fracture toughness enhancement in ceramic matrix composites through tailored interfaces.

The potential benefits that may be derived from functionally graded composites have led to increased activities in the areas of processing, and materials science, of these materials. However, these activities are seriously handicapped by the lack of appropriate computational strategies for the response of functionally graded materials that explicitly couple the heterogeneous microstructure of the material with the global analysis. The standard micromechanics approach used to analyze the response of this class of materials is to decouple the local and global effects by assuming the existence of a representative volume element (RVE) at every point within the composite, Figure 1 (cf., Yamanouchi, 1990; Wakashima and Tsukamoto, 1990; Fukushima, 1992). This assumption, however, neglects the possibility of coupling between local and global effects, thus leading to potentially erroneous results in the presence of macroscopically nonuniform material properties and large field variable gradients. This is particularly true when the temperature gradient is large with respect to the dimension of the inclusion phase, the characteristic dimension of the inclusion phase is large relative to the global dimensions of the composite, and the number of uniformly or nonuniformly distributed inclusions is relatively small (Aboudi et al., 1993). Perhaps the most important objection to using the standard RVE-based micromechanics approach in the analysis of FGMs is the lack of a theoretical basis for the definition of an RVE, which clearly *cannot be unique in the presence of continuously changing properties due to nonuniform inclusion spacing*.

As a result of the limitation of the standard micromechanics approaches, a new higher-order micromechanical theory for functionally graded materials (**HOTFGM**), that explicitly couples the local and global effects, has been developed. This theory allows coupled micro-macromechanical analysis of composite plates functionally graded in the through-thickness direction that are subjected to a thermal gradient in the same direction (Aboudi et al., 1993; 1994a,b). The development of the theory has been justified by comparison with the results obtained using the standard micromechanics approach which neglects the micro-macrostructural coupling effects explicitly taken into account in the new theory (Pindera et al., 1994, 1995). Due to the absence of such coupling, the standard micromechanics approach often *underestimates* actual stress distributions in composites with a finite number of uniformly or nonuniformly distributed fibers across the thickness dimension subjected to a thermal gradient.

A limited comparison has been presented by Goldberg and Hopkins (1995) between the predictions of **HOTFGM** and the results of a boundary-element analysis of thermal fields in composites with a finite number of through-thickness rows of fibers subjected to a thermal gradient. This comparison partially establishes the reliability of the coupled higher-order theory in accurately predicting thermal fields in the presence of temperature gradients. Herein, extensive comparison between the predictions of the finite-element method and **HOTFGM** are presented for both the thermal and stress fields in unidirectional SiC/Ti composites subjected to a through-thickness temperature gradient. This comparison establishes the theory as an accurate tool in the analysis and optimization of functionally graded architectures in metal matrix composites.

## **HOTFGM: A COUPLED HIGHER-ORDER THEORY FOR FGMS**

**HOTFGM** is based on the geometric model of a heterogeneous composite, with a finite thickness  $H$ , extending to infinity in the  $x_2-x_3$  plane and subjected to a temperature gradient produced by the temperature  $T_T$  and  $T_B$  applied to the top and bottom surfaces, respectively, Figure 2. The composite is reinforced by periodic arrays of fibers in the direction of the  $x_2$  axis or the  $x_3$  axis, or both. In the direction of the  $x_1$  axis, called the functionally gradient (**FG**) direction, the fiber spacing between adjacent arrays may vary. The reinforcing fibers can be either continuous or finite-length. The heterogeneous composite is constructed using a generic unit cell, Figure 3, which consists of either four or eight subcells, depending on whether continuously or discontinuously reinforced functionally graded composites are considered. The generic unit cell in the present framework is not taken to be an RVE whose effective properties can be obtained through homogenization (Hill, 1963). Rather, the RVE comprises an entire column of such cells spanning the plate's thickness. Thus the response of each cell is explicitly coupled to the response of the entire column of cells in the **FG** direction, thereby directly coupling the microstructural

details with the global analysis. This is in stark contrast with the standard uncoupled micromechanics approach commonly used in the analysis of FGMs.

The solution to the thermo-mechanical boundary-value problem outlined in the foregoing is solved in two steps. In the first step, the temperature distribution in a single column of cells, representative of the composite-at-large, spanning the FG dimension is determined by solving the heat equation under steady-state conditions in each cell subject to the appropriate continuity and compatibility conditions. The solution to the heat equation is obtained by approximating the temperature field in each  $(\alpha\beta\gamma)$  subcell of a generic unit cell using a quadratic expansion in the local coordinates  $\bar{x}_1^{(\alpha)}$ ,  $\bar{x}_2^{(\beta)}$ ,  $\bar{x}_3^{(\gamma)}$ , centered at the subcell's mid-point, that reflects the microstructure's periodicity and symmetry in the  $x_2$  and  $x_3$  directions (i.e., absence of linear terms in the local coordinates  $\bar{x}_2^{(\beta)}$  and  $\bar{x}_3^{(\gamma)}$ ):

$$T^{(\alpha\beta\gamma)} = T_0^{(\alpha\beta\gamma)} + \bar{x}_1^{(\alpha)} T_1^{(\alpha\beta\gamma)} + \frac{1}{2}(3\bar{x}_1^{(\alpha)2} - \frac{d_\alpha^{(p)2}}{4})T_2^{(\alpha\beta\gamma)} + \frac{1}{2}(3\bar{x}_2^{(\beta)2} - \frac{h_\beta^2}{4})T_3^{(\alpha\beta\gamma)} + \frac{1}{2}(3\bar{x}_3^{(\gamma)2} - \frac{l_\gamma^2}{4})T_4^{(\alpha\beta\gamma)} \quad (1)$$

A higher-order representation of the temperature field is necessary in order to capture the local effects created by the thermomechanical field gradients, the microstructure of the composite and the finite dimension in the FG direction, in contrast with previous treatments involving fully periodic composite media (Aboudi, 1991). The unknown coefficients associated with each term in the expansion are then obtained by constructing a system of equations that satisfies the requirements of a standard boundary-value problem for the given temperature field approximation. That is, the heat equation is satisfied in a volumetric sense, and the thermal and heat flux continuity conditions within a given cell, as well as between a given cell and adjacent cells, are imposed in an average sense. This yields  $40M$  equations in the unknown  $40M$  coefficients  $T_i^{(\alpha\beta\gamma)}$  for a composite with  $M$  rows of fibers in the through-thickness direction of the form:

$$\kappa \mathbf{T} = \mathbf{t} \quad (2)$$

where the structural thermal conductivity matrix  $\kappa$  contains information on the geometry and thermal conductivities of the individual subcells  $(\alpha\beta\gamma)$  in the  $M$  cells spanning the thickness of the FG plate, the thermal coefficient vector  $\mathbf{T} = (T_1^{(111)}, \dots, T_M^{(222)})$ , where  $T_p^{(\alpha\beta\gamma)} = (T_0, T_1, T_2, T_3, T_4)_p^{(\alpha\beta\gamma)}$ , contains the unknown coefficients that describe the thermal field in

each subcell, and the thermal force vector  $\mathbf{t} = (T_T, 0, \dots, 0, T_B)$  contains information on the thermal boundary conditions. The details of derivation of the above system of equations are given in Aboudi et al. (1993, 1994b).

Given the temperature distribution in a single column of cells representative of the composite-at-large, internal displacements, strains and stresses are subsequently generated by solving the equilibrium equations in each cell subject to appropriate continuity and boundary conditions. The solution is obtained by approximating the displacement field in the FG direction in each subcell using a quadratic expansion in local coordinates within the subcell. The displacement field in the  $x_2$  and  $x_3$  directions, on the other hand, is approximated using linear expansion in local coordinates that reflects the periodic character of the composite's microstructure in the  $x_2$  and  $x_3$  directions.

$$u_1^{(\alpha\beta\gamma)} = w_1^{(\alpha\beta\gamma)} + \bar{x}_1^{(\alpha)} \phi_1^{(\alpha\beta\gamma)} + \frac{1}{2}(3\bar{x}_1^{(\alpha)2} - \frac{1}{4}d_\alpha^{(\beta)2})U_1^{(\alpha\beta\gamma)} + \frac{1}{2}(3\bar{x}_2^{(\beta)2} - \frac{1}{4}h_\beta^2)V_1^{(\alpha\beta\gamma)} \quad (3)$$

$$+ \frac{1}{2}(3\bar{x}_3^{(\gamma)2} - \frac{1}{4}l_\gamma^2)W_1^{(\alpha\beta\gamma)}$$

$$u_2^{(\alpha\beta\gamma)} = \bar{x}_2^{(\beta)} \chi_2^{(\alpha\beta\gamma)} \quad (4)$$

$$u_3^{(\alpha\beta\gamma)} = \bar{x}_3^{(\gamma)} \psi_3^{(\alpha\beta\gamma)} \quad (5)$$

As a consequence of the chosen displacement field representation, the present formulation leads to a description of a functionally graded composite whose overall deformation is characterized by the vanishing of the average composite strains  $\bar{\epsilon}_{22}$  and  $\bar{\epsilon}_{33}$ . This follows directly from eqns (4) and (5) which contain no constant terms that represent subcell center displacements. It is possible to generalize the present theory by including subcell center displacements that produce uniform strains  $\bar{\epsilon}_{22}$  and  $\bar{\epsilon}_{33}$ . This generalization leads to an overall behavior of a composite, functionally graded in the  $x_1$  direction, which can be described as a generalized plane strain in the  $x_1$ - $x_2$  and  $x_1$ - $x_3$  planes (Aboudi et al., 1995a).

The unknown coefficients associated with each term in the expansion, i.e.,  $w_1^{(\alpha\beta\gamma)}$ ,  $\phi_1^{(\alpha\beta\gamma)}$ ,  $\chi_2^{(\alpha\beta\gamma)}$ ,  $\psi_3^{(\alpha\beta\gamma)}$ ,  $U_1^{(\alpha\beta\gamma)}$ ,  $V_1^{(\alpha\beta\gamma)}$ ,  $W_1^{(\alpha\beta\gamma)}$ , are obtained by satisfying the appropriate field equations in a volumetric sense (0-th, 1-st and 2-nd moment), together with the boundary conditions and continuity of displacements and tractions between individual subcells of a given cell, and between adjacent cells. The continuity conditions are imposed in an average sense. This results in  $56M$  equations in the unknown  $56M$  coefficients in the displacement representation for each cell for a

composite with  $M$  rows of fibers in the through-thickness direction of the form:

$$\mathbf{K} \mathbf{U} = \mathbf{f} \quad (6)$$

where the structural stiffness matrix  $\mathbf{K}$  contains information on the geometry and thermo-mechanical properties of the individual subcells ( $\alpha\beta\gamma$ ) in the  $M$  cells spanning the thickness of the FG plate. The displacement coefficient vector  $\mathbf{U}$  contains the unknown coefficients that describe the displacement field in each subcell, i.e.,

$$\mathbf{U} = (\mathbf{U}_1^{(111)}, \dots, \mathbf{U}_M^{(222)}) \quad (7)$$

where  $\mathbf{U}_p^{(\alpha\beta\gamma)} = (w_1, \phi_1, U_1, V_1, W_1, \chi_2, \psi_3)_p^{(\alpha\beta\gamma)}$ , and the mechanical force vector  $\mathbf{f}$  contains information on the mechanical boundary conditions and the thermal loading effects generated by the applied temperature. The details of derivation of the above system of equations are given in Aboudi et al. (1993, 1994b).

## FINITE-ELEMENT ANALYSIS

The comparison between finite-element and **HOTFGM** predictions was carried out for three plate configurations with one, three and five uniformly-spaced rows of SiC fibers in the thickness direction embedded in a titanium matrix. The **HOTFGM** results have been previously reported by Aboudi et al. (1993) for configurations in which the first row of fibers was located directly adjacent to the top surface (i.e., the matrix layer between the first row of fibers and the top surface was removed). These configurations were subjected to a through-thickness temperature gradient of 500°C by maintaining the top surface at 0°C and the bottom surface at 500°C. The top surface was traction-free whereas the bottom surface was constrained by imposing zero displacement in the thickness direction.

The geometry of the basic unit cell used in the finite-element analysis to construct these configurations is shown in Figure 4. As in the **HOTFGM** model, the apex of the fiber is flush with the upper surface of the unit cell. The dimensions of the unit cell for the given fiber radius produce a composite with a fiber volume fraction of 0.40. These dimensions are one third of the actual dimensions employed in Aboudi et al. (1993). However, since the problem is linearly elastic the actual dimensions do not matter so long as the proper geometric ratios are maintained, which is the situation here. The boundary conditions employed in the finite-element analysis for the configuration with one row of fibers through the plate's thickness are also shown in the

figure. These boundary conditions simulate a state of plane strain along the  $x_2$  direction.

The material properties of the SiC fibers and the titanium matrix employed in the analysis are given in Table 1. For the purpose of the comparison presented herein, the SiC fibers and the titanium matrix were treated as elastic with temperature-independent properties. The thermal conductivity mismatch between the constituents was deliberately amplified (i.e.,  $\kappa_f / \kappa_m = 50$ ) in order to critically test the predictive capability of the new theory.

The finite-element results were generated using the commercially-available finite-element code ABAQUS (1989). This was carried out by first performing heat transfer analysis to determine the temperature distribution throughout the analyzed region, and subsequently using the nodal temperatures as input in the mechanical analysis. In order to ensure convergence of the thermal and stress fields, successively finer meshes were constructed and the results obtained with each refined mesh compared with those obtained with the preceding mesh. The initial mesh for the basic unit cell was constructed with 66 elements, Figure 5a, while the final mesh which produced satisfactorily convergent thermal and stress fields consisted of 248 elements, Figure 5b. For the thermal analysis, two-dimensional heat transfer, three- and four-noded (DC2D3 and DC2D4, respectively), linear elements were employed. For the mechanical analysis, plane strain three- and four-noded linear elements (CPE3 and CPE4, respectively) were employed in order to account for the vanishing average strain field in the  $x_3$  direction obtained from **HOTFGM**. The refined meshes employed to simulate configurations with three and five rows of fibers through the plate's thickness are shown in Figure 6.

## COMPARISON OF HOTFGM AND FINITE-ELEMENT RESULTS

The comparison between finite-element and **HOTFGM** predictions for the temperature and normal stress  $\sigma_{22}$  and  $\sigma_{33}$  through-thickness distributions are presented in the fiber/matrix and matrix/matrix cross-sections (see Figure 4). The fiber/matrix cross-section passes through the diametral plane of each fiber along the  $x_1$  direction, whereas the matrix/matrix cross-section lies halfway between adjacent fibers along the  $x_2$  direction. The various distributions along the thickness direction are given as a function of the normalized coordinate  $x_1 / M$ , where  $M$  is the number of fiber rows. The maximum value of  $x_1 / M$  is 199  $\mu\text{m}$ , which corresponds to the thickness of a single layer employed in Aboudi et al. (1993). The corresponding fiber diameter used in the above study which produces fiber volume fraction of 0.40 is 142  $\mu\text{m}$ . Since the results generated by ABAQUS were based on a single layer thickness of 66.33  $\mu\text{m}$  and fiber diameter of 47.34  $\mu\text{m}$ , the locations of the actual points at which the field quantities were evaluated were scaled up by a factor of three to bring them into geometric correspondence with the **HOTFGM** results obtained by Aboudi et al. (1993).

Figure 7 presents comparison between **HOTFGM** and finite-element results for the temperature distributions in a SiC/Ti plate with one, three and five rows of fibers in the fiber/matrix (Figure 7a) and matrix/matrix (Figure 7b) representative cross-sections (RCS). In the RCS containing both phases, the temperature profiles exhibit pronounced "staircase" patterns, characterized by small temperature gradients in the fiber phase and much larger gradients in the matrix phase. This is consistent with the large thermal conductivity of the SiC fiber employed in the present calculations relative to that of the matrix. Virtually no difference is observed between the results of the higher-order theory and finite-element analysis in the fiber/matrix cross-section. It is worthwhile to point out that the same results were obtained independently by Goldberg and Hopkins (1995) in the fiber-matrix cross-section of a configuration with three rows of fibers through the plate's thickness using the boundary-element method. In this study, however, only the thermal fields were considered.

Alternatively, the temperature profiles predicted by **HOTFGM** in the RCS containing only matrix generally exhibit smoother transitions between fiber and matrix phases away from the boundaries (i.e., in the interior) relative to the fiber/matrix cross-section, without an apparent staircase pattern caused by the presence of adjacent fibers. In contrast, the influence of adjacent fibers is observed in the finite-element results which exhibit small oscillations about the **HOTFGM** predictions. These oscillations resemble the staircase patterns seen in the fiber-matrix cross-section temperature profiles, with substantially smoother transitions, however, between the matrix regions adjacent to the fibers and the matrix regions adjacent to the matrix in the fiber-matrix cross-section. The differences between the higher-order theory and finite-element results in the matrix-matrix RCS are generally small, and further these differences decrease with increasing number of fibers through the plate's thickness.

The corresponding normal stress  $\sigma_{22}$  and  $\sigma_{33}$  distributions are presented in Figures 8 and 9, respectively, in both characteristic cross-sections. In the case of the RCS containing both phases, Figures 8a and 9a, the stress profiles predicted by **HOTFGM** exhibit characteristic patterns characterized by jumps at the fiber/matrix interfaces when the number of fibers through the plate's thickness is greater than one. These jumps occur because the normal stresses  $\sigma_{22}$  and  $\sigma_{33}$  are not traction components associated with the fiber/matrix interfaces normal to the  $x_1$  direction. Substantially smaller stress gradients are observed in the fiber than in the matrix phase, as suggested by the corresponding temperature profiles. The finite-element results closely match the higher-order theory predictions, with somewhat greater differences observed in the  $\sigma_{22}$  distributions than in the  $\sigma_{33}$  distributions. In fact, the  $\sigma_{33}$  distributions obtained from the finite-element calculations are virtually the same as the higher-order theory predictions for the configurations with three and five rows of fibers through the plate's thickness. In contrast, the  $\sigma_{22}$

distributions obtained from the finite-element calculations exhibit some (generally small) departures from the higher-order theory predictions. The presence of abrupt stress gradient changes and stress oscillations in the matrix phase observed in the finite-element predictions for  $\sigma_{22}$  suggests that the observed differences may be eliminated by further mesh refinement.

In the case of the RCS containing matrix only, Figures 8b and 9b, the  $\sigma_{22}$  and  $\sigma_{33}$  stress profiles predicted by **HOTFGM** also exhibit jumps at elevations corresponding to the fiber/matrix interfaces in the adjacent fiber-matrix cross-sections. These jumps are typically greater in the  $\sigma_{22}$  distributions since this stress component must be continuous along the  $x_2$  axis, and thus is influenced to a greater extent by the corresponding stress in the adjacent fiber-matrix cross-sections than  $\sigma_{33}$ . Since only one material is present in the matrix-matrix cross-section, the stress gradients away from the boundaries do not change significantly in regions separated by planes passing through the fiber/matrix interfaces where the jumps occur. As in the case of the stress distributions in the fiber-matrix cross-section, generally good agreement is observed between the higher-order theory and finite-element results, with better correlation for the  $\sigma_{33}$  distribution than the  $\sigma_{22}$  distribution. The greatest differences occur in the matrix regions adjacent to the fibers in the neighboring fiber-matrix cross-sections. These differences are due to the sharp spike-like profiles observed in the finite-element results, suggesting that further mesh refinement in those regions may potentially improve the correlation. In contrast, the differences between the higher-order theory and finite-element predictions in the matrix regions adjacent to the matrix phase in the neighboring cross-sections are quite small for both stress distributions.

## CONCLUSIONS AND FUTURE PERSPECTIVES

This report presented a comparison of the thermal and stress fields in a composite plate subjected to a thermal gradient generated using a recently developed coupled higher-order theory for functionally graded materials and the finite-element analysis. In this new approach, the microstructural and macrostructural details are *explicitly coupled* when analyzing the response of a composite subjected to a uniform or nonuniform loading such as a through-thickness temperature gradient. Coupling of the local and global analyses is required to rationally analyze the response of polymeric and metal matrix composites such as B/Ep, B/Al and SiC/TiAl that contain relatively few through-thickness fibers, as well as the newly emerging functionally graded materials with continuously changing properties due either to nonuniform fiber spacing or the presence of several phases. For such materials, it is difficult, if not impossible, to define the representative volume element used in the traditional micromechanical analyses of macroscopically homogeneous composites.

The comparison between the predictions of the coupled higher-order theory and finite-element analysis has been presented for a SiC/TiAl composite plate reinforced by one, three and five equally-spaced rows of fibers through the plate's thickness and subjected to a through-thickness 500°C temperature gradient. The presented comparison demonstrates that the proposed theory is an accurate and efficient method for investigating internal temperature and stress fields in composites with a finite number of fibers in the thickness direction which cannot be analyzed using the standard micromechanics approach based on the concept of a representative volume element. It is indeed remarkable that the higher-order theory's predictions obtained using a relatively coarse discretization of the composite's microstructure compare very favorably with the finite-element results obtained using a very finely discretized mesh. In fact, considerable effort has been made in the finite-element mesh refinement process to obtain satisfactorily convergent thermal and stress fields. Conversely, only four subcells per generic unit cell were required to generate higher-order theory results of comparable accuracy. Additional validation of the accuracy of the higher-order theory in predicting thermal fields in composites with a finite number of through-thickness rows of fibers subjected to a thermal gradient has been provided recently by Goldberg and Hopkins (1995) using the boundary-element method. The results presented herein, reinforced by the Goldberg and Hopkins data, set the stage for the application of the higher-order theory to the newly emerging class of functionally graded composites having continuously varying microstructures with confidence.

The recent enhancements of the higher-order theory's capabilities through the addition of inelastic constitutive models (Aboudi et al., 1995b) make possible the investigation of functionally graded composite plates subjected to through-thickness thermal gradients in a wide temperature range. In particular, functional grading of metallic layers protected by ceramic thermal barrier coatings has been shown to reduce the resulting thermally-induced warping tendency in such configurations. Such applications of the higher-order theory open up new areas of research dealing with the concept of optimum thermal management through functionally graded material architectures. Along similar lines, the most recent extension of the one-dimensional version of the theory to materials functionally graded in two directions makes possible the investigation of the potential of functionally graded fiber architectures in reducing edge effects in laminated composites (Aboudi et al., 1995c).

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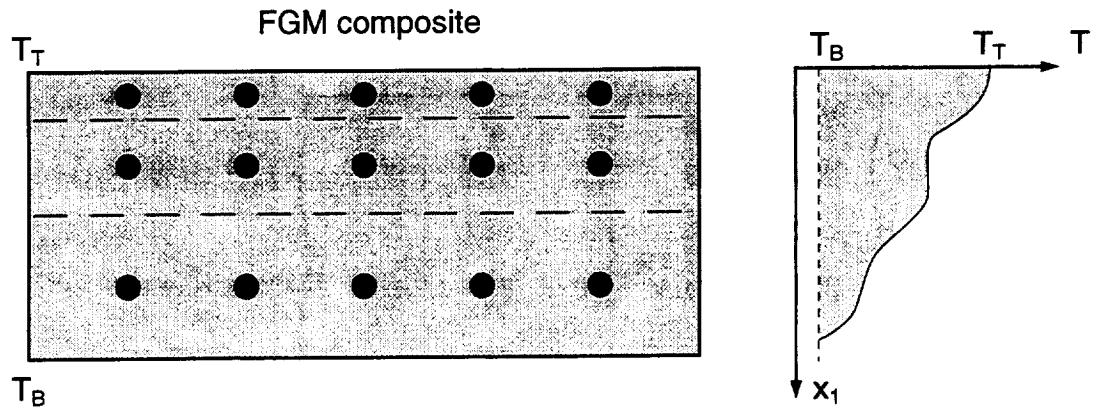
## REFERENCES

- ABAQUS User's Guide, v4.8 (1989). Hibbit, Karlsson & Sorensen, Inc., Pawtucket, RI.
- Aboudi, J. (1991), *Mechanics of Composite Materials - A Unified Micromechanical Approach*, Elsevier, New York.
- Aboudi, J., Pindera, M-J., and Arnold, S. M. (1993), "Thermoelastic Response of Metal Matrix Composites with Large-Diameter Fibers Subjected to Thermal Gradients", *NASA TM 106344*, NASA Lewis, Cleveland, OH.
- Aboudi, J., Arnold, S. M., and Pindera, M-J. (1994a), "Response of Functionally Graded Composites to Thermal Gradients," *Composites Engineering*, Vol. 4, No. 1, pp. 1-18.
- Aboudi, J., Pindera, M-J., and Arnold, S. M. (1994b), "Elastic Response of Metal Matrix Composites with Tailored Microstructures to Thermal Gradients," *International Journal of Solids and Structures*, Vol. 31, No. 10, pp. 1393-1428.
- Aboudi, J., Pindera, M-J., and Arnold, S. M. (1995a), "A Coupled Higher-Order Theory for Functionally Graded Materials with Partial Homogenization," *Composites Engineering* (in press).
- Aboudi, J., Pindera, M-J., and Arnold, S. M. (1995b), "Thermo-Inelastic Response of Functionally Graded Composites," *International Journal of Solids and Structures* (in press).
- Aboudi, J., Pindera, M-J., and Arnold, S. M. (1995c), "Thermoelastic Theory for the Response of Materials Functionally Graded in Two Directions with Applications to the Free-Edge Problem," *NASA TM 106882*, NASA Lewis, Cleveland, OH.
- Fukushima, T. (Ed). (1992), *Proceedings of the International Workshop on Functionally Gradient Composites*, November 5-6, 1992, Hilton Square, San Francisco, USA. Japan International Science and Technology Exchange Center.
- Goldberg, R. K. and Hopkins, D. A. (1995), "Thermal Analysis of a Functionally Graded Material Subject to a Thermal Gradient Using the Boundary Element Method," *Composites Engineering* (in press).
- Hill, R. (1963), "Elastic Properties of Reinforced Solids: Some Theoretical Principles," *J. Mech. Phys. Solids*, Vol. 11, pp. 357-372.
- Pindera, M-J., Aboudi, J., and Arnold, S. M. (1994), "Thermo-Inelastic Analysis of Functionally Graded Materials: Inapplicability of the Classical Micromechanics Approach," in *Inelasticity and Micromechanics of Metal Matrix Composites*, G. Z. Voyatzis and J. W. Ju (Eds.), pp. 273-305, Elsevier Science B.V., Amsterdam, The Netherlands.
- Pindera, M-J., Aboudi, J., and Arnold, S. M. (1995), "Limitations of the Uncoupled, RVE-Based Micromechanical Approach in the Analysis of Functionally Graded Composites," *Mechanics of Materials* (in press).
- Yamanouchi, M., Koizumi, M., Hirai, T., and Shiota, I. (Eds). (1990), *Proceedings of the First International Symposium on Functionally Gradient Materials*, Sendai, Japan.
- Wakashima, K. and Tsukamoto, H. (1990), "Micromechanical Approach to the Thermomechanics of Ceramic-Metal Gradient Materials," *Proc. First International Symposium on Functionally Gradient Materials*, pp. 19-26, Sendai, Japan.

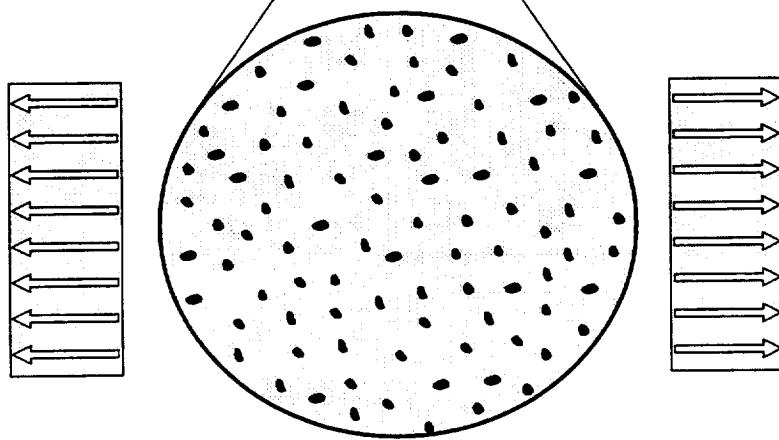
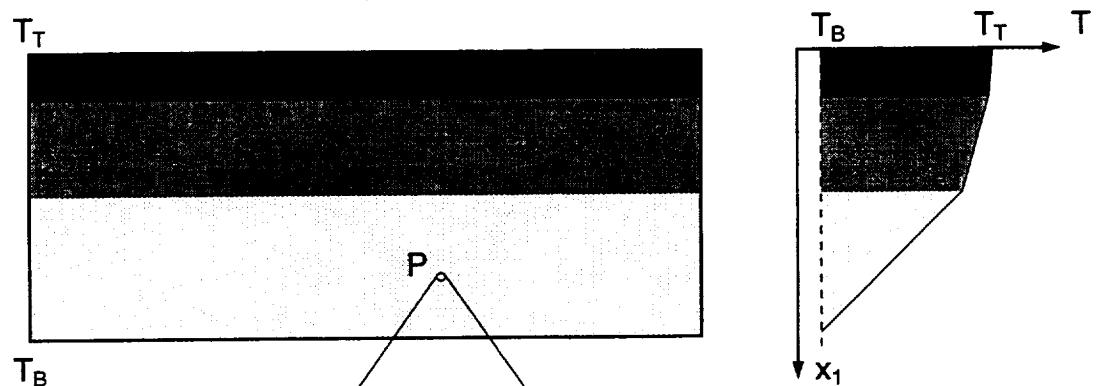
Table 1. Material properties of SCS-6 SiC fiber and titanium matrix.

| Material     | E (GPa) | $\nu$ | $\alpha (10^{-6} \text{ m / m / } ^\circ\text{C})$ | $\kappa (\text{W / m-}^\circ\text{C})$ |
|--------------|---------|-------|--|--|
| SiC fiber    | 414.0   | 0.3   | 4.9  | 400.0                                  |
| Ti-Al matrix | 100.0   | 0.3   | 9.6  | 8.0                                    |

E and  $\nu$  denote the Young's modulus and Poisson's ratio, respectively,  $\alpha$  is the coefficient of thermal expansion, and  $\kappa$  is the thermal conductivity.

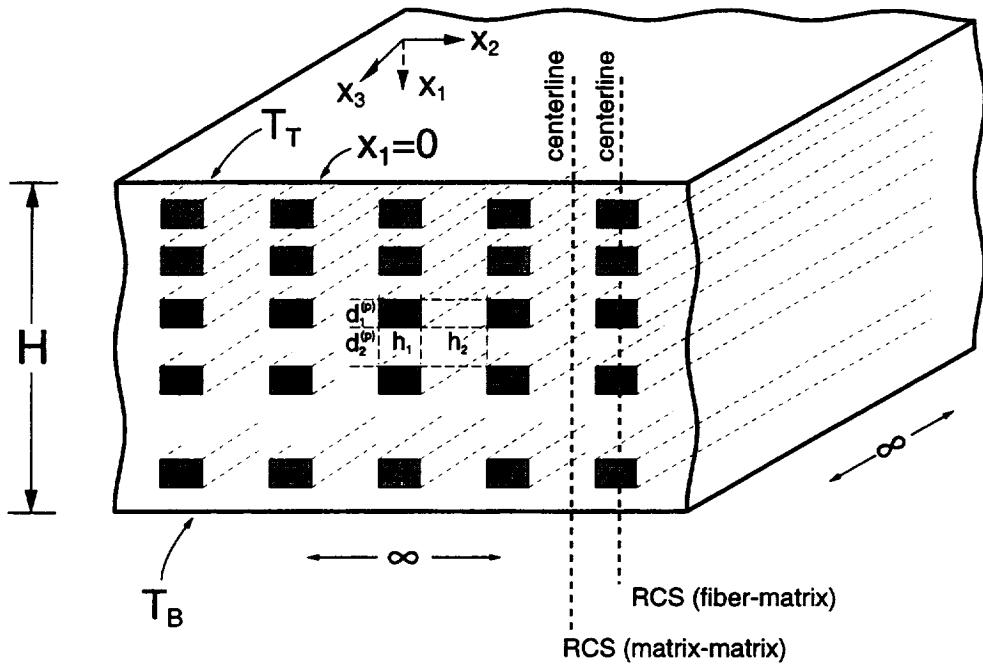


Equivalent continuum with piece-wise uniform effective properties

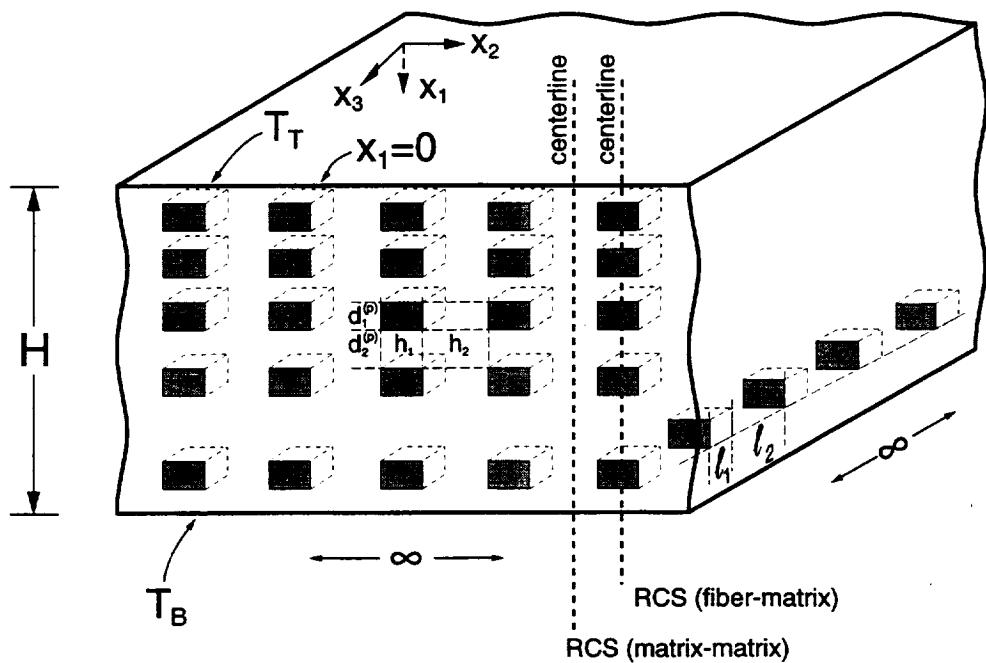


An RVE subjected to homogeneous boundary conditions to determine effective properties at point P.

Figure 1. Uncoupled micromechanics analysis of FGMS.



a.)



b.)

Figure 2. Composite with nonperiodic fiber distribution in the  $x_1$  direction: a) unidirectionally reinforced material; b) particulate inclusion reinforced material. RCS denotes the representative cross-section.

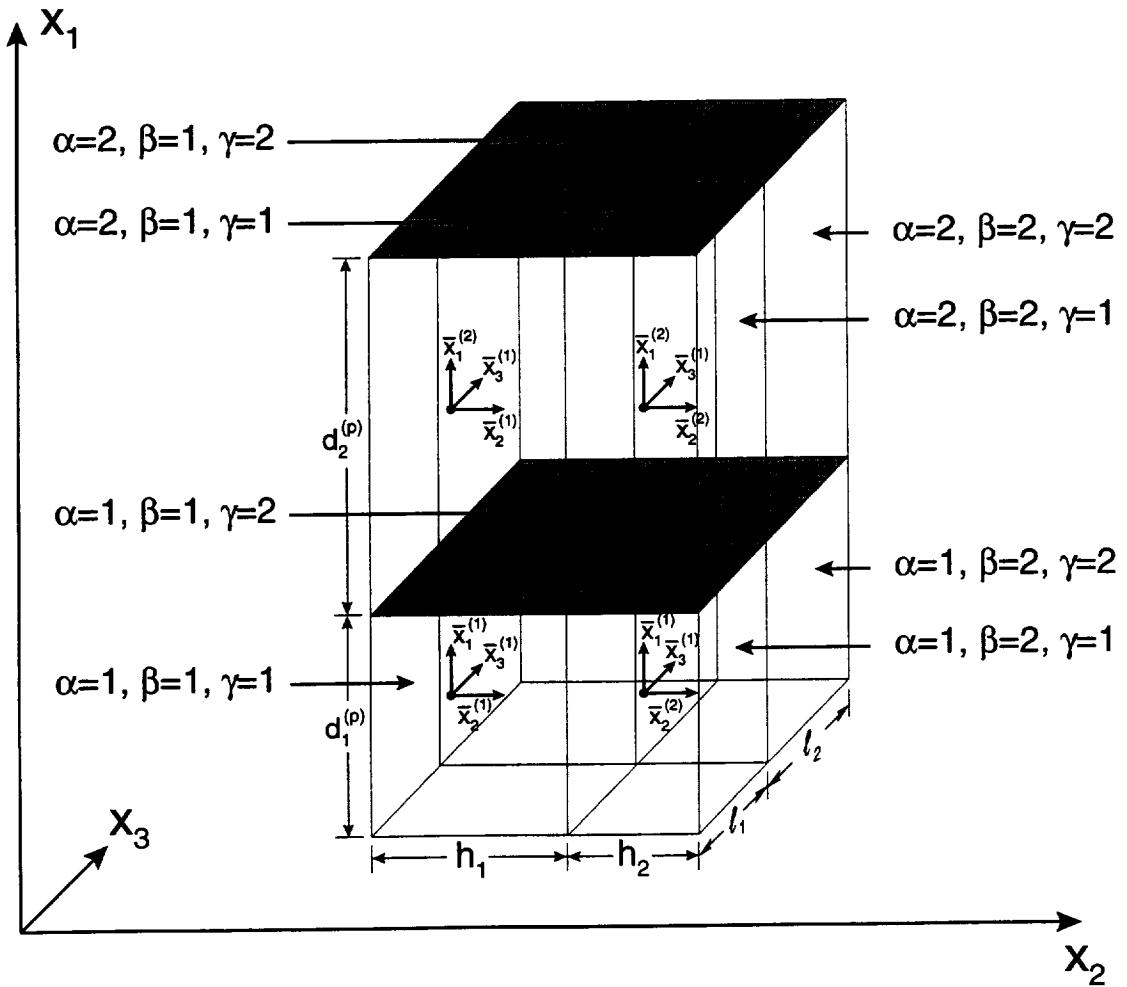


Figure 3. The generic unit cell of a composite with nonperiodic fiber distribution in the  $x_1$  direction.

**Section 1-1**  
fiber/matrix  
cross section

**Section 2-2**  
matrix/matrix  
cross section

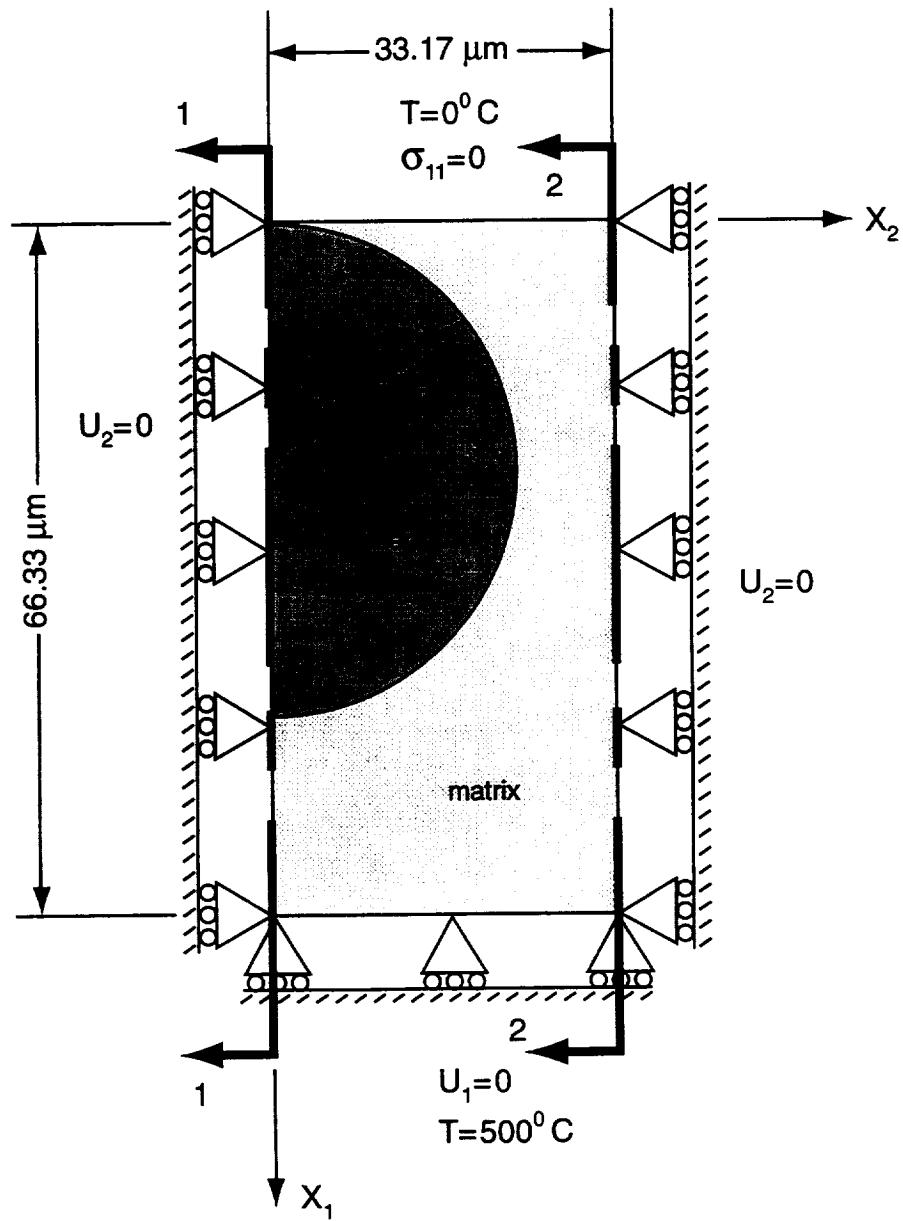
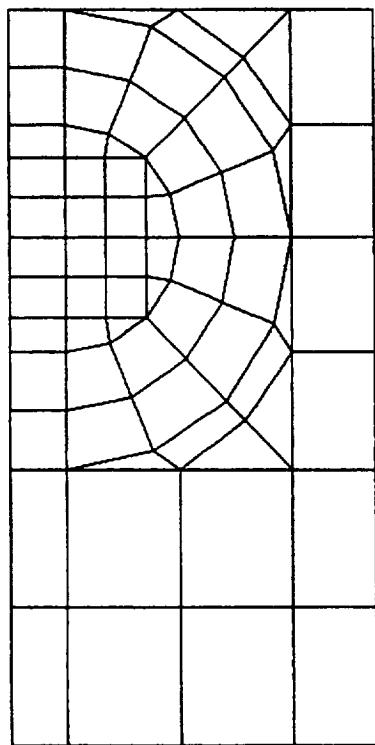
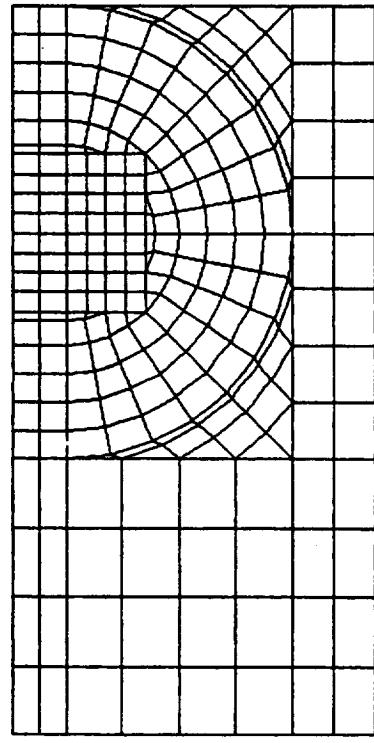


Figure 4. Geometry of the basic unit cell used in the finite-element analysis and the boundary conditions for the configuration with one row of fibers through the plate's thickness.

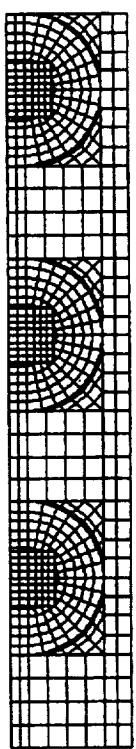


a)

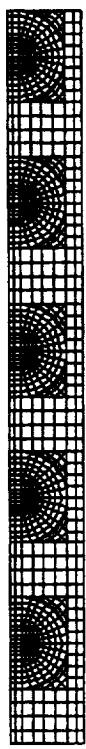


b)

Figure 5. Details of the mesh for the configuration with one row of fibers through the plate's thickness used in the finite-element analysis: a) initial mesh; b) refined mesh.

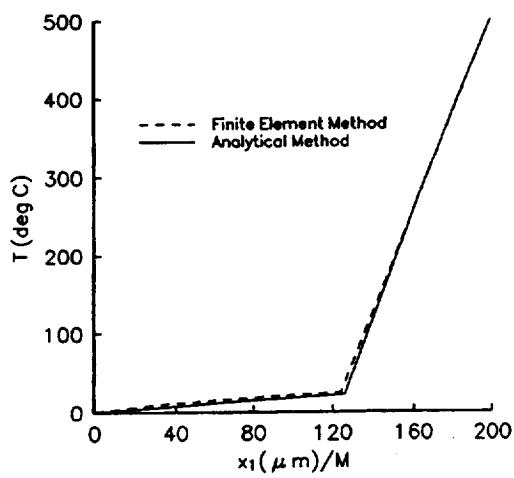


a)

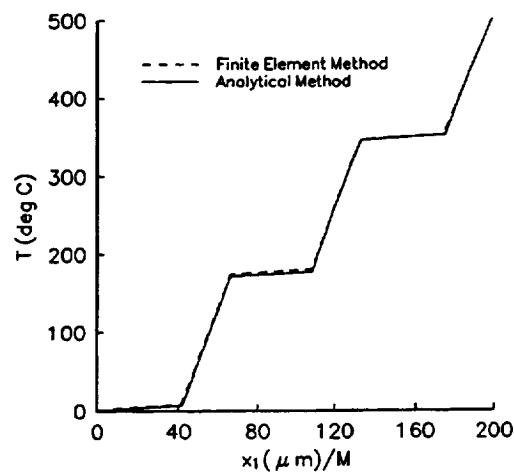
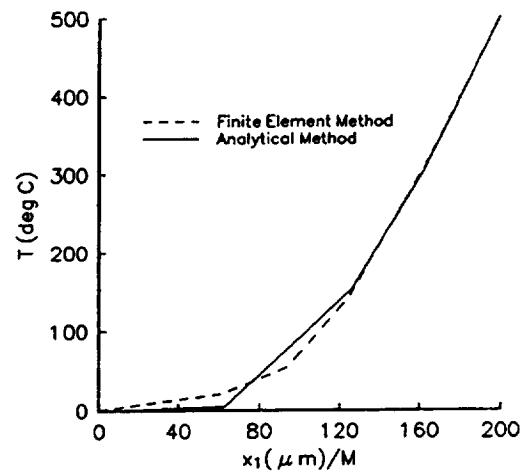


b)

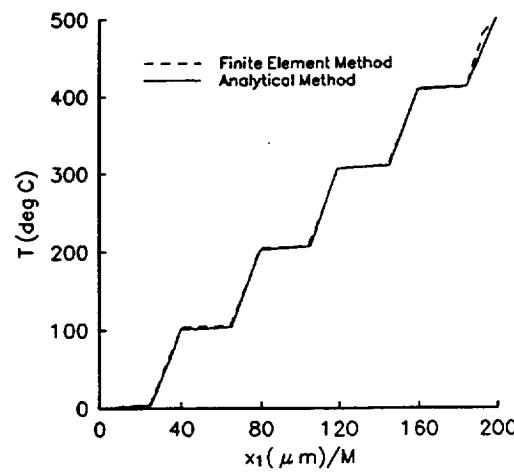
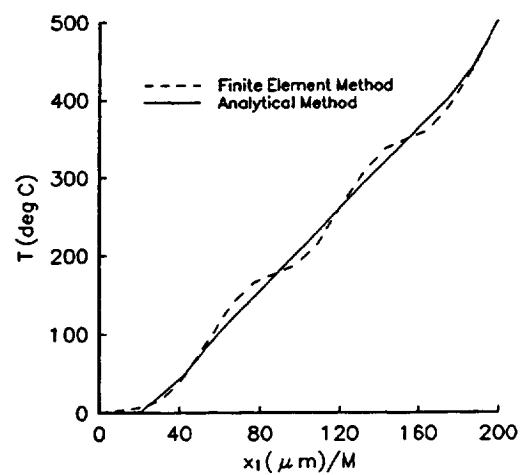
Figure 6. Details of the refined meshes for the configurations with three (a) and five (b) rows of fibers through the plate's thickness used in the finite-element analysis.



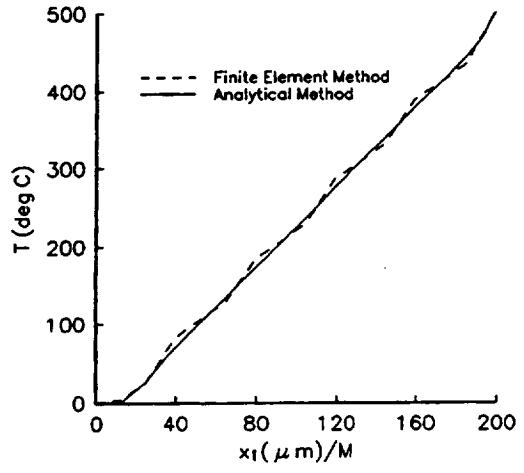
$M = 1$



$M = 3$



$M = 5$



a)

b)

Figure 7. Comparison between the coupled higher-order theory and the finite-element analysis of the thermal fields in a SiC/Ti composite with one, three and five uniformly-spaced fibers in the thickness direction: a) temperature profile across the fiber-matrix RCS; b) temperature profile across the matrix-matrix RCS.

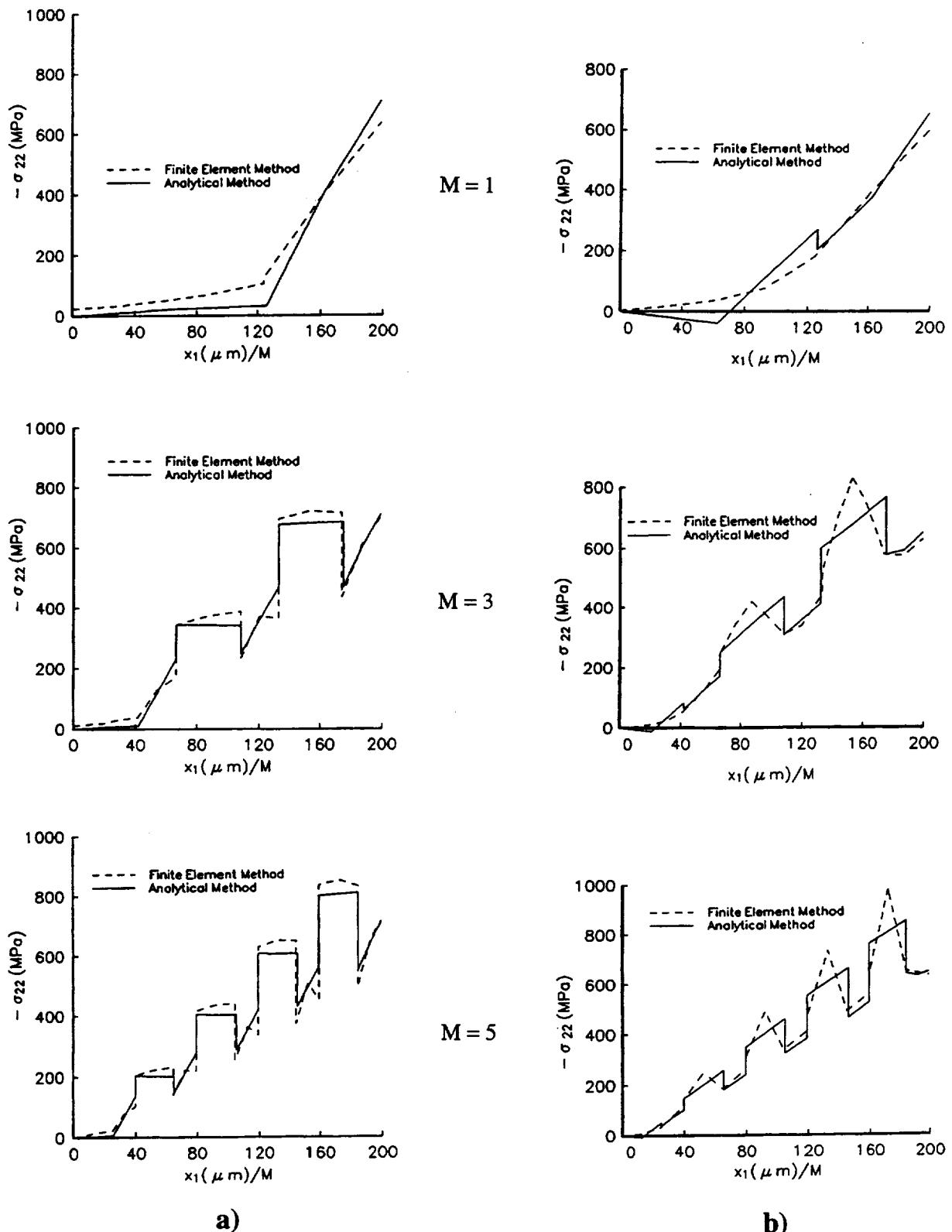


Figure 8. Comparison between the coupled higher-order theory and the finite-element analysis of the normal  $\sigma_{22}$  stress in a SiC/Ti composite with one, three and five uniformly-spaced fibers in the thickness direction: a) stress profile across the fiber-matrix RCS; b) stress profile across the matrix-matrix RCS.

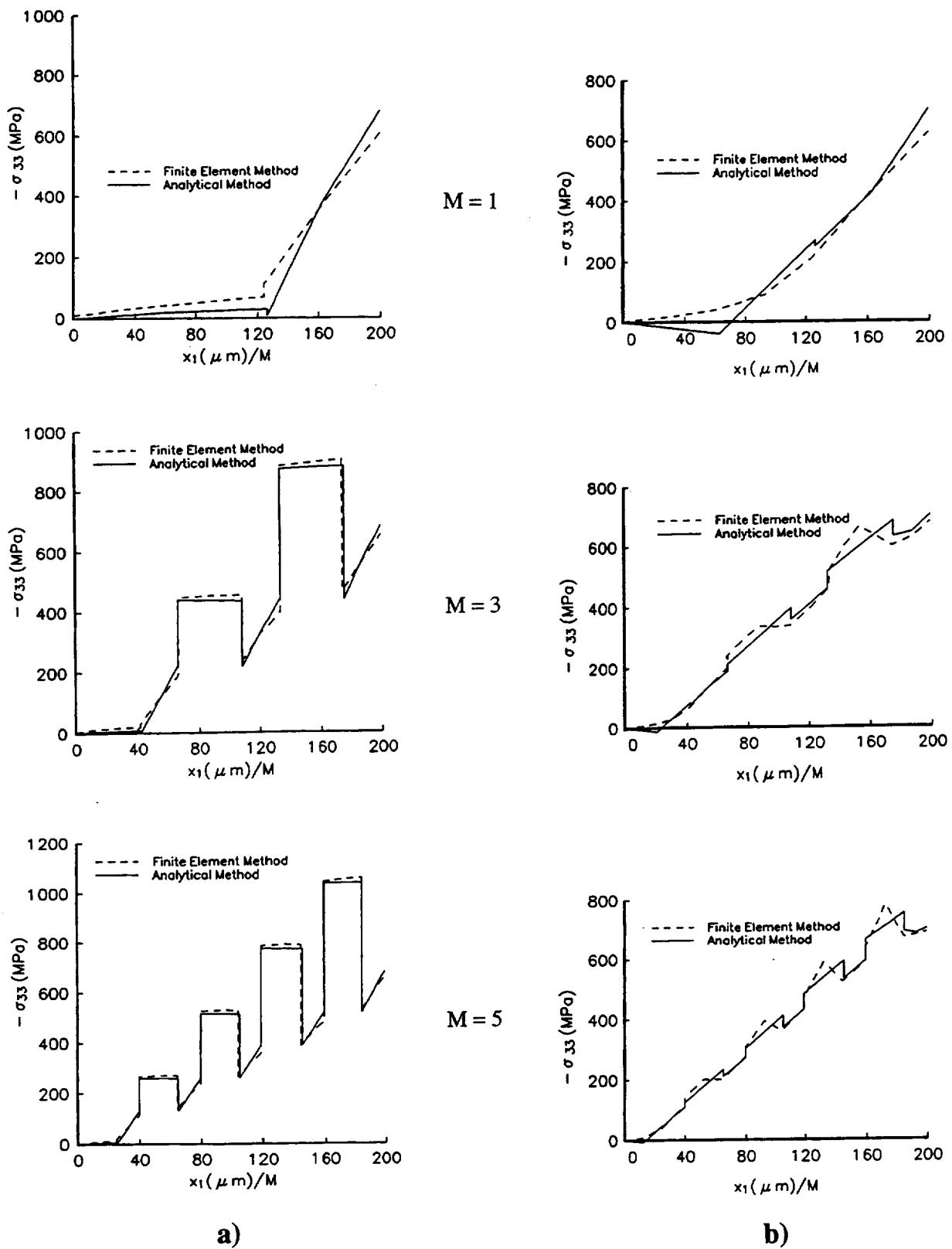


Figure 9. Comparison between the coupled higher-order theory and the finite-element analysis of the normal  $\sigma_{33}$  stress in a SiC/Ti composite with one, three and five uniformly-spaced fibers in the thickness direction: a) stress profile across the fiber-matrix RCS; b) stress profile across the matrix-matrix RCS.

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